

# Mathematical Modeling with R: Embedding Computational Thinking into High School Math Classes

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# Teaching **R** to high school students **is possible**

To the regular math student: i.e., not enrolled in computer science classes

Youngsters with diverse career interests

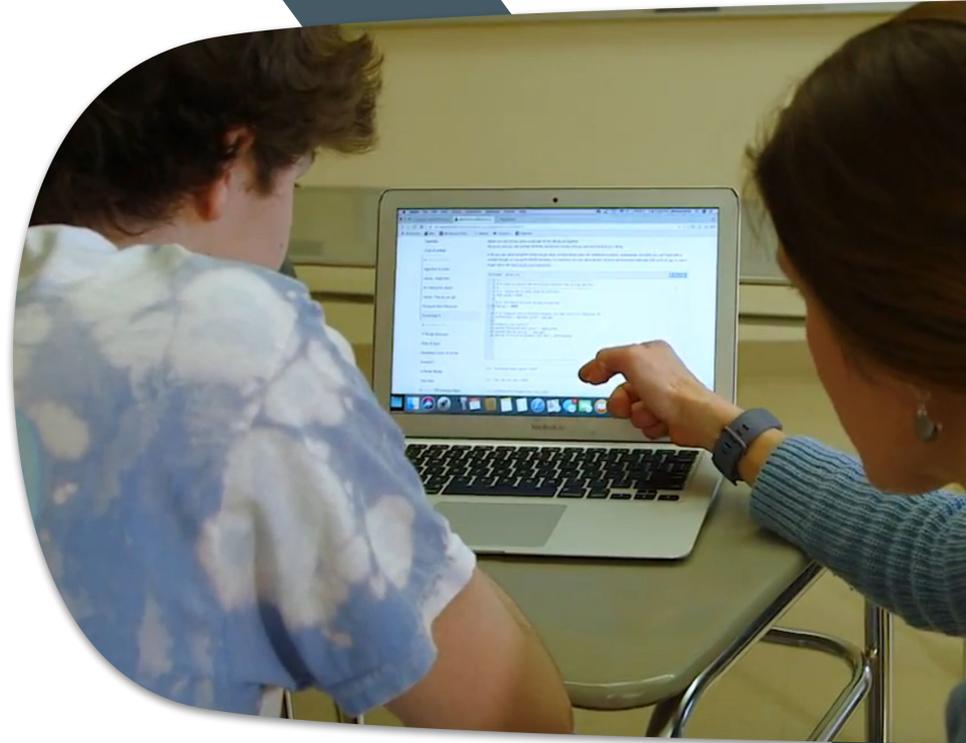
Integrating computational thinking into regular math classrooms:

Mathematical Modeling <<>> Programming

Using **R** as the **modeling environment**

# Overview

- Curriculum module and assessment
- Quick view of the pilot results
- Strategies that made it work
  - Context-Based Instruction
  - Faded Scaffolding
  - Removing Syntax-Related Stress



# Lifehacking module

## Solving practical problems

### Learning sequence:

Activity: Meal Plan vs. Pay As You Go

» High to low scaffolding

### Assessment:

Activity: Driving for Gas

» **No** scaffolding

## Dining at UMass

Let's consider the particular case of the University of Massachusetts Amherst (UMass). On the next two pages, we outline some essential information about meal plans (for dining commons) and pay as you go (for retailers) and some links for you to find additional information.

While reading about the two options, think about the following questions. Discuss with your partners and jot down your ideas.

- What matters to college students when it comes to dining?
- Does it make sense to compare the costs of two options?
- What assumptions do you have to make to justify that it is a fair comparison?



Retailers at the Campus Center



The dining common

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# Lifehacking module

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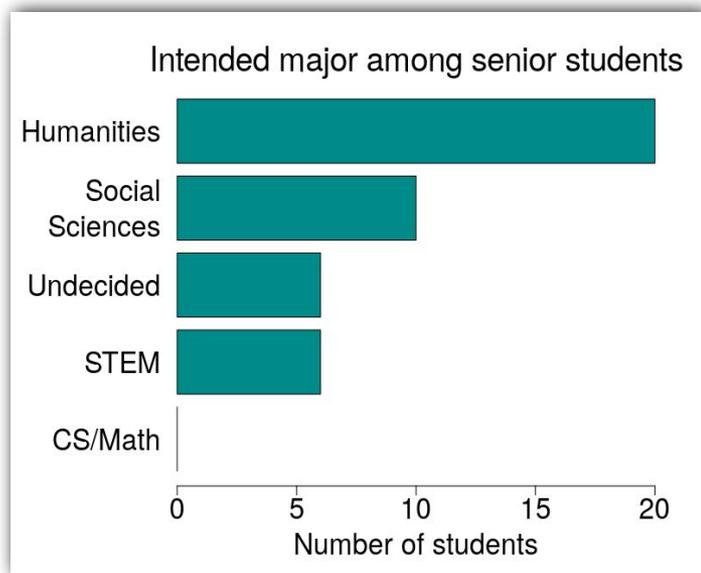
Activity: Driving for Gas

» Little/No scaffolding

```
# MODEL -----  
# Gas Station 1 (GS1) is right where the driver is.  
# Gas Station 2 (GS2) requires a detour.  
# The prices of regular gas at GS1 and GS2 (dollar/gallon)  
price_gs1 = 3.00  
price_gs2 = 2.75  
  
# The amount of gas the driver needs to buy at either station (gallon)  
amount = 15  
  
# The costs of buying gas at GS1 or GS 2 (dollar)  
cost_gs1 = price_gs1 * amount  
cost_gs2 = price_gs2 * amount  
  
# The additional distance for the detour to GS2 (mile)  
distance_add = 1:30  
  
# The fuel economy of the driver's car (miles per gallon)  
mpg = 25  
  
# The amount of gas used for the detour to GS2 (gallon)  
amount_add = distance_add / mpg  
  
# The additional cost for the detour to GS2 (dollar)  
cost_add = price_gs2 * amount_add  
  
# The real cost of buying gas at GS2, taking the detour into consideration  
cost_gs2_real = cost_gs2 + cost_add  
  
# Compare the costs of buying gas at GS1 versus GS2  
difference = cost_gs1 - cost_gs2_real  
  
# REPORT RESULTS -----  
# Display the data generated by the model  
data.frame(distance_add, cost_gs2_real, difference)  
  
# Impact of additional distance on the real cost of buying gas at GS2  
plot(x = distance_add, y = cost_gs2_real, ylim = c(30, 50))  
  
# Visualize the cost of buying gas at GS1 as a horizontal line  
abline(h = cost_gs1)
```



# The pilot sample



- 42 senior students
- 2nd semester of last year in high school
- Ages 17 - 18
- Regular mathematics class
- No prior experience with coding
- Never enrolled in computer science classes
- 'Left-tail' math achievers
- Social sciences & humanities-inclined
- Not really interested either in mathematics or coding

# Lifehacking module

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Variables

Vectors

Operators

Functions

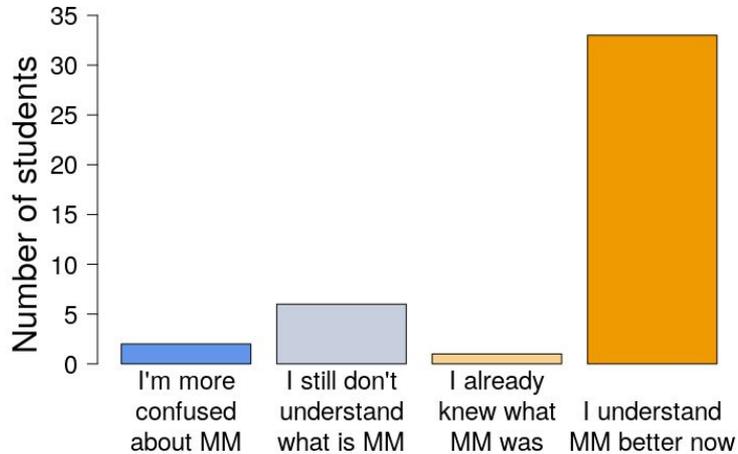
Data types

Modeling process



# Using R as Math Modeling Environment

Understanding of Mathematical Modeling  
after **6 hours** of modeling activity with R



**80%** of students said they now understand mathematical modeling better

**One-third** said they became interested in taking programming classes in the future

*"I just found it really fascinating how coding can make life that much easier!"*



*"It opened up my mind to other ways in which math can be used in the real world."*

# CodeR4MATH Platform

Equity and Access

R Markdown  
learnr  
Shiny



Self-paced tutorials  
Reduced syntax-stress  
Online access

# Clear, step-by-step instructions in a self-paced tutorial



## Snacks?

Something important is missing! The students need snacks to keep their energy up. In the R coding space below, incorporate snacks to your estimate of the daily cost of pay-as-you-go. Consider creating a variable for this new item to make it easy to update its estimate later.

- Add code to the existing code below.
- Run the new code and check the output.
- Click **Hint** if you need help.
- Done? Ask the teacher to check your work.

```
Code Start Over Hint Run Code
1 # show menu
2 head(menu)
3 # create breakfast cost
4 breakfast = mean(bakery$price) + max(beverage$price)
5 # create lunch cost
6 lunch = quantile(entree$price, probs = 0.75) + mean(beverage$price)
7 # create dinner cost
8 dinner = max(salad$price) + min(beverage$price)
9 # create a snack cost
10 snack = median(bakery$price)
11 # estimate daily cost
12 daily = breakfast + snack + lunch + dinner + snack
13 #report daily cost
14 paste("Daily cost:", round(daily), "dollars!")
15
```

name	type	retailer	size	price	sales
<fctr>	<fctr>	<fctr>	<fctr>	<dbl>	<int>
1 fountain drink	beverage	bamboo	N	1.0	128
2 daily lunch and dinner	entree	bamboo	N	9.0	24
3 dim sum 2 pieces	entree	bamboo	S	3.5	90
4 dim sum six pieces	entree	bamboo	L	9.0	23
5 fountain drink	beverage	deli_delish	N	1.0	195
6 blue wall club	sandwich	deli_delish	N	8.0	66

6 rows

[1] "Daily cost: 36 dollars!"

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Snippets that can be  
ran, modified, restarted

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Hints and occasional solutions that students can check at any time

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Data is pre-loaded and  
outputs are right next  
to the code

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# Strategies that made it work

Context-Based Instruction

Faded Scaffolding

Removed Syntax-Related Stress

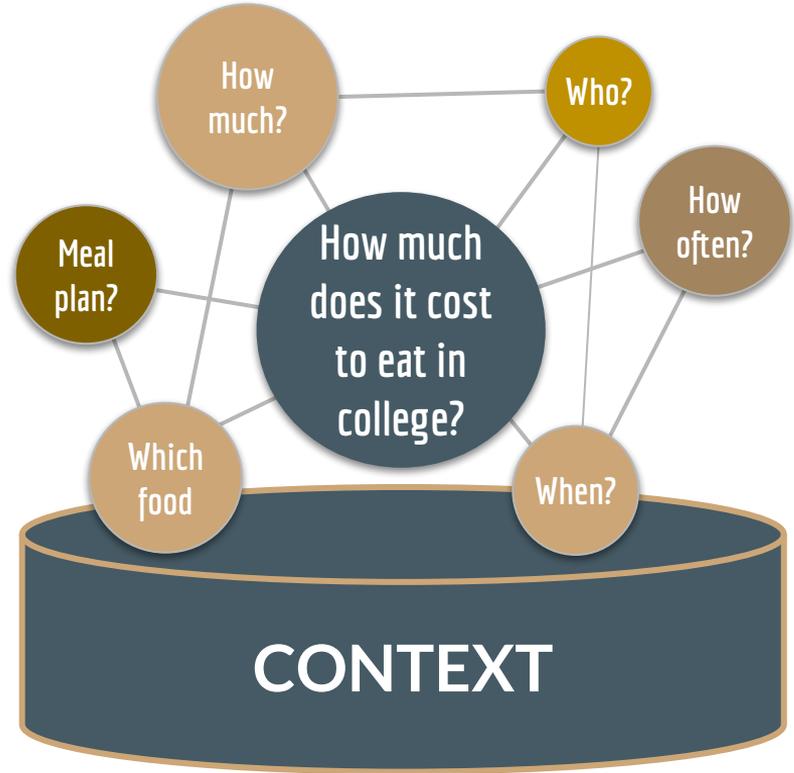
# Context-Based

- Start with an open-ended problem
- Everything is taught in the context of that initial problem

## MODEL

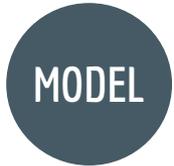
Make assumptions  
Choose parameters  
Build an algorithm  
Hypothesize outcomes

- To test your model under a variety of circumstances **you will want a code**



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Question: What is the impact of one day off campus? And 2 days? And 3? 4? And so on...

$\text{total.cost} = \text{semester cost} - 1 * \text{daily.cost}$

$\text{total.cost} = \text{semester.cost} - 2 * \text{daily.cost}$

$\text{total.cost} = \text{semester.cost} - 3 * \text{daily.cost}$

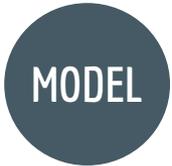
$\text{total.cost} = \text{semester.cost} - 4 * \text{daily.cost}$

$\text{total.cost} = \text{semester.cost} - 5 * \text{daily.cost}$

- 
- 
-

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- 
- 
- 

$$\text{days.off} = 1:40$$

$$\text{total.cost} = \text{semester.cost} - \text{days.off} * \text{daily.cost}$$

**YAY!**

**A vector!**

**What a great idea!**



# Faded scaffolding

High: The problem is already solved:  
*Run the snippet, see the output*

Medium: The problem is partially solved:  
*Modify the snippet, run it again,  
compare with previous results*

Low: e.g., create new parameters.

Learning **concepts** (e.g. variables, vectors)  
and **tools** (e.g. functions)

What option is less expensive?  
How much is the difference between the two options?

Let's pretend that we already know which meal plan to choose and how much the total cost of pay-as-you-go would be. We can write an R code to represent this end result.

- Run the code and check the output.
- Change 3269 on the first line to 3067 (to reflect a different meal plan) and run the code again. See the new outputs?
- To restore the original code, click **Start Over** on the top-left corner.

```
Code Start Over Hint Solution Submit! Run Code  
1 # Choose which mealplan to consider  
2 meal_plan = 3269  
3  
4 # Estimate the cost of pay-as-you-go  
5 pay_go = 3000  
6  
7 # Compute the difference between the two  
8 difference = abs(meal_plan - pay_go)  
9  
10 # Report the results  
11 paste("Difference between the two options: $", difference)
```

```
[1] "Selected meal plan: 3269"
```

```
[1] "Pay as you go: 3000"
```

```
[1] "Difference between the two: 269"
```

In the following pages, we will recreate this code snippet together, step by step. You will learn the programming concepts behind it and be able to customize it to your liking.

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Prior to this point, we assigned \$8 to the `lunch` variable. Let's replace this number with the expression above (see the code below). So when the counselors work with their students, they can provide more accurate estimates based on the students' situations. For instance, very healthy eaters may just want a salad for lunch while athletes may add sides for more calories.

Code

```
1 # Assign 'lunch' with the sum of average sandwich price and average beverage price
2 lunch = min(sandwich$price) + max(beverage$price) + min(dessert$price)
3
4 # Report the cost of lunch
5 paste("Cost of lunch:", lunch)
```

[1] "Cost of lunch: 13.23"

In the coding space below, compose your meals and estimate their costs using statistical functions and the addition operator `+`, and assign the estimates to the `breakfast`, `lunch`, `dinner`, and `snack` variables. If you have done the first part in *Activity 3 - Cost of a Meal*, copy and paste your code here and go from there.

Code

```
1 # Estimate the cost of breakfast
2
3 # Estimate the cost of lunch
4
5 # Estimate the cost of dinner
6
7 # Estimate the cost of snack
```

# Conclusion

# It works!

- Teaching R to pre-college students is **possible and worthwhile!**
- R is **powerful** and yet **friendly** enough to be taught at high school
- **Context** and faded scaffolding boosts the learning process
- **Reducing syntax-stress** pays off
- Social Sciences and Humanities-inclined students **can and should** learn the value of math modeling and coding

# New Platform (under development)

- Exercises & Assessments
- Teacher dashboard
- Data collection
- Educational and research-oriented



STEM Resource Finder

## CodeR4MATH

CodeR4MATH provides a robust path for integrating math and computing learning. We have developed a learning platform integrating R computing resources, curriculum materials, automated assessment and tutoring, and teacher professional development resources.

Create an account to start using this material with your students.

[Sign Up](#)

Projects

**Units**

Activities

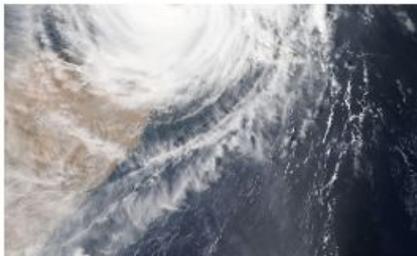
Assessments

R Tutorials



### Unit 1: Lifehacking

Model to solve practical problems, such as the costs of eating in college or owning and driving a car.



### Unit 2: Earth Sciences

Let's gather environmental data and see how scientists model weather events to help policy-makers.



### Unit 3: Engineering for life

Are solar panels worthy? Recycle or compost? Reducing costs and carbon footprint with math modeling!

Collections

About

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Computing with R for Mathematical Modeling (CodeR4MATH).

resources, curriculum materials, automated assessment and tutoring, and teacher professional development resources.



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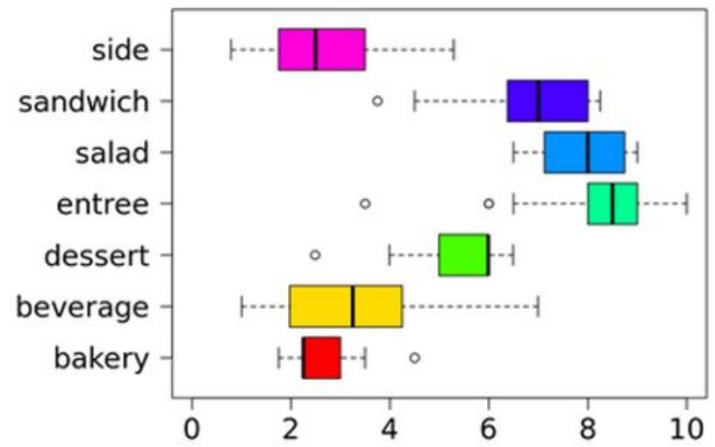
Phasellus vel lacus volutpat, dapibus leo eget, malesuada massa. Sed eget arcu at turpis luctus tempus vitae ut dui.

### Meal Plan vs. Pay As You Go

- 1 Introduction
- Math Modeling
- About R
- R Coding Space
- My Code
- 2 The Challenge
- Overloaded Counselors
- Dining at UMass
- Dining Commons
- Retailers
- Assumptions
- 3 Retailers' Menu

To make it more appealing, let's add some color. Use the `col` argument and use the `rainbow()` to generate 7 rainbow colors.

```
Code Start Over Run Code
1 boxplot(menu$price ~ menu$type,
2   col = rainbow(7), horizontal = T, las = 1, ylin = c(0,10))
3
```



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Merci!



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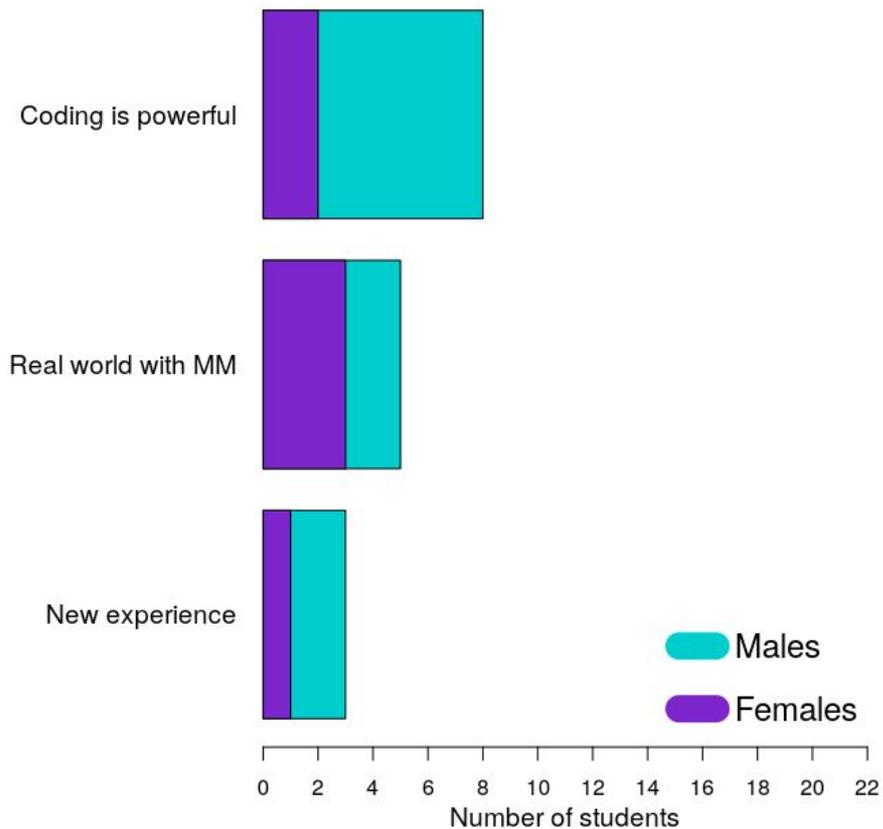


Kenia Wiedemann  
Concord Consortium  
kwiedemann@concord.org





## Students who got INTERESTED in taking programming classes



## Students still NOT interested in taking programming classes

